Date

MATH2230 Complex Variables with Application
Suggested Solution for HW 11

SEC. 94.

1. Solution: $C: |z|=1$

(a) Let $f(z)=z^2$. Then $Z=2$, $P=0$.

$\Delta_C \arg f(z) = 2\pi(Z-P) = 4\pi$.

(b) Let $f(z)=\frac{1}{z^2}$. Then $Z=0$, $P=2$.

$\Delta_C \arg f(z) = 2\pi(Z-P) = -4\pi$

(c) Let $f(z)=\frac{(2z-1)^7}{z^3}$. Then $Z=7$, $P=3$.

$\Delta_C \arg f(z) = 2\pi(Z-P) = 8\pi$.

Remark: Check the conditions (a)(b) of Thm. in SEC. 93 by yourself.

2. Solution: Obviously, $\Delta_C \arg f(z) = 2\pi \times 3 = 6\pi$ from the figure.

By the Thm. in SEC. 93, we have $Z-P=3$.

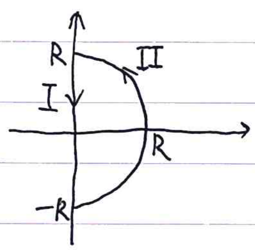
Since f is analytic in $\mathbb{C}$, we have $P=0$.

Thus, $Z=3$.

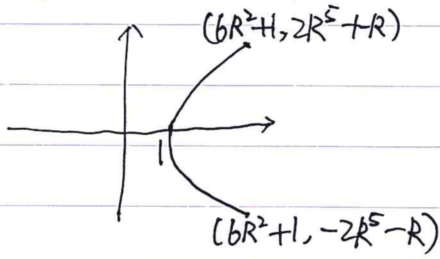
Therefore, the number of zeros is 3.

Q3. Find how many roots does the function $f(z)=2z^5-6z^2+z+1$ have on the right-half plane.

Solution: I: ti, $t: R \to -R$



$$f(z) = 2(ti)^5 - 6(ti)^2 + ti + 1 = 2it^5 + 6t^2 + ti + 1$$
$$= (6t^2+1) + i(2t^5+t)$$



change of argument is

$$\arctan\frac{-2R^5-R}{6R^2+1} - \arctan\frac{2R^5+R}{6R^2+1} = -2\arctan\frac{2R^5+R}{6R^2+1} \to -\pi$$

as $R\to\infty$

II: $Re^{i\theta}$, $\theta: -\frac{\pi}{2} \to \frac{\pi}{2}$

$$f(z) = 2R^5e^{i5\theta} - 6R^2e^{i2\theta} + Re^{i\theta} + 1 = R^5(2e^{i5\theta} - 6R^{-3}e^{i2\theta} + R^{-4}e^{i\theta} + R^{-5})$$

As $R\to\infty$, change of argument is 5π

Thus, the total change of argument is 4π. Therefore, number of roots is $\frac{4\pi}{2\pi}=2$